Interactive Markov Chains using CADP

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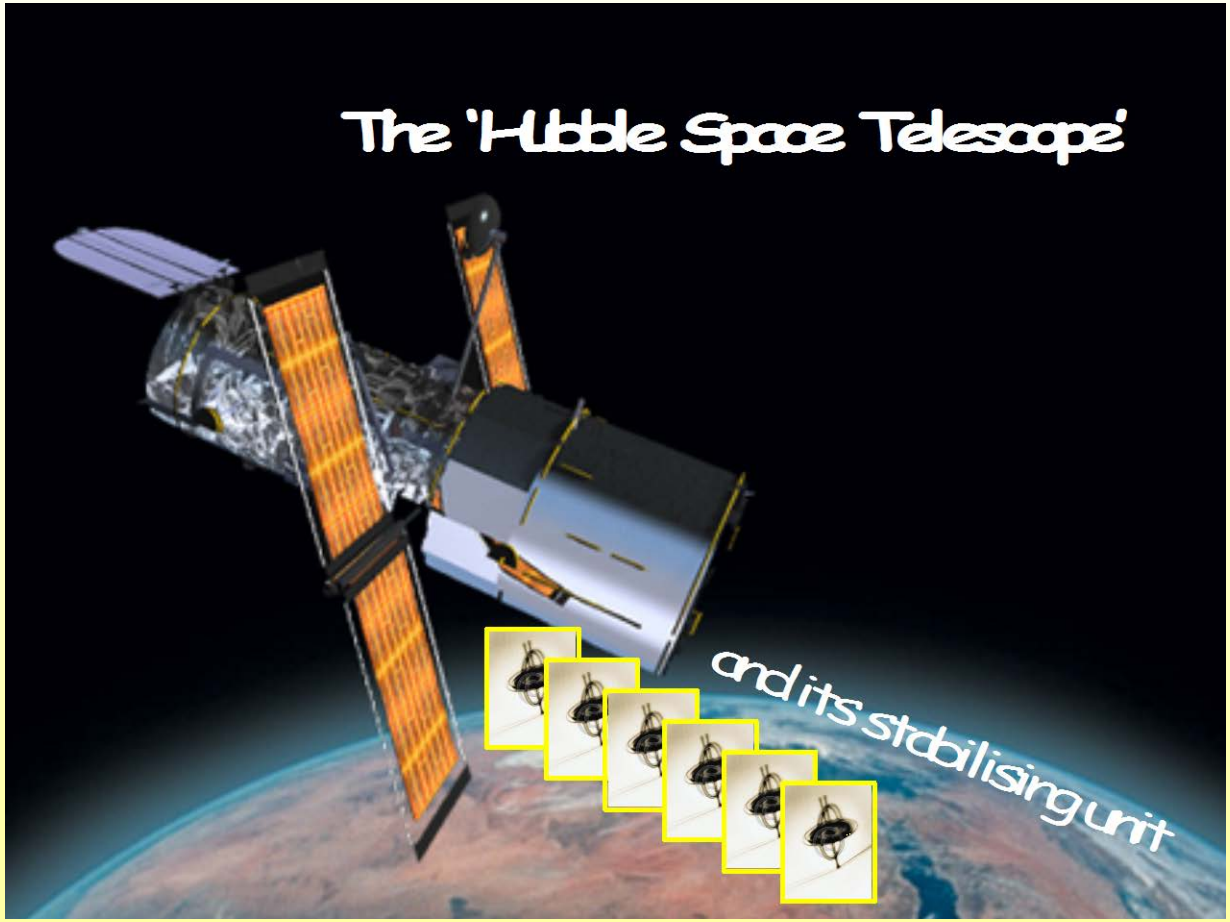
How to predict the lifetime of the Hubble telescope?

Hubert Garavel (based on H. Hermanns' work)

The Hubble space telescope



A simple Markov model for the Hubble

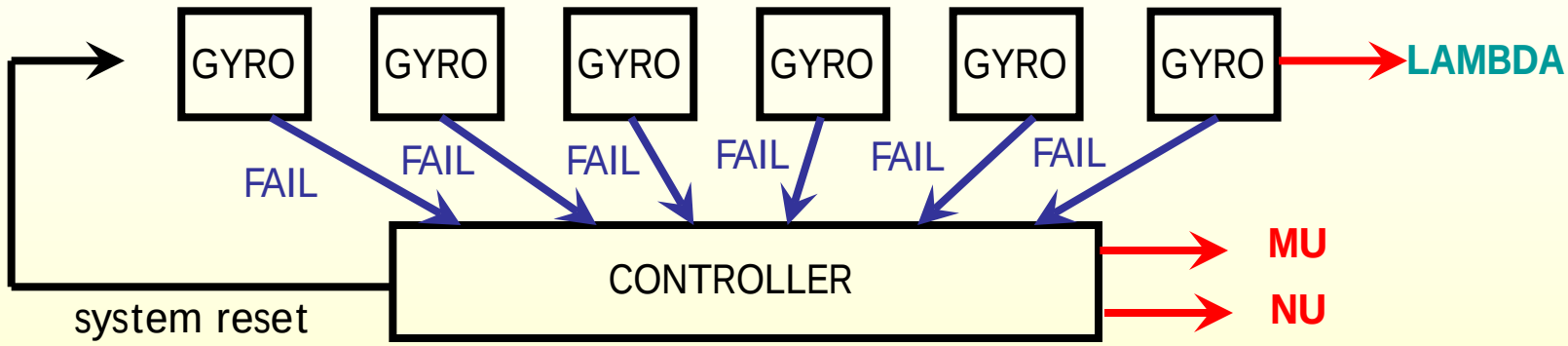
- The Huble telescope has 6 gyroscopes
- As time passes, the gyros may fail
- The average lifetime of gyros is 10 years (= 120 months)

  λ = *12 months / 120 = 0.1*
- Hubble falls into sleep if only two gyros are left
- Turning on sleep mode requires to halt all equipments, which takes about 3.6 days (= 0.12 month)

  μ = *12 months / 0.12 = 100*
- When in sleep mode, a shuttle mission must be sent to repair/reset Hubble, which takes about 2 months

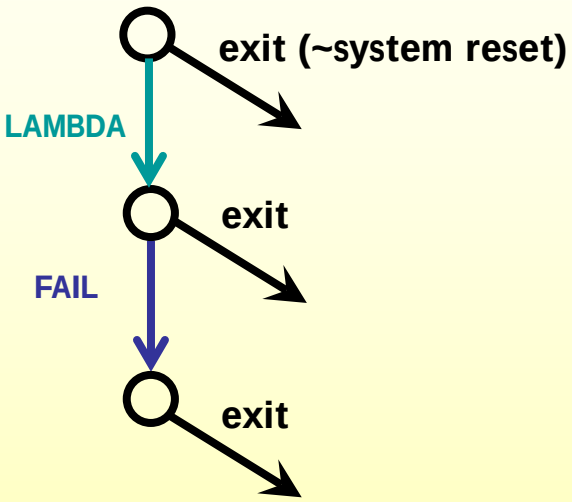
  ν =*12 months / 2 = 6*
- Without operational gyro, Hubble crashes

Compositional modelling of the Hubble




```
process HUBBLE [LAMBDA, MU, NU] : noexit :=
    hide FAIL in
      (
          (
          GYRO [LAMBDA, FAIL] ||| GYRO [LAMBDA, FAIL] ||| GYRO [LAMBDA, FAIL]
|||
          GYRO [LAMBDA, FAIL] ||| GYRO [LAMBDA, FAIL] ||| GYRO [LAMBDA, FAIL]
          )
        |[FAIL]|
        CONTROLLER [FAIL, MU, NU] (6, false)
        >> (* system reset *)
        HUBBLE [LAMBDA, MU, NU]
      )
endproc
```

The GYRO process

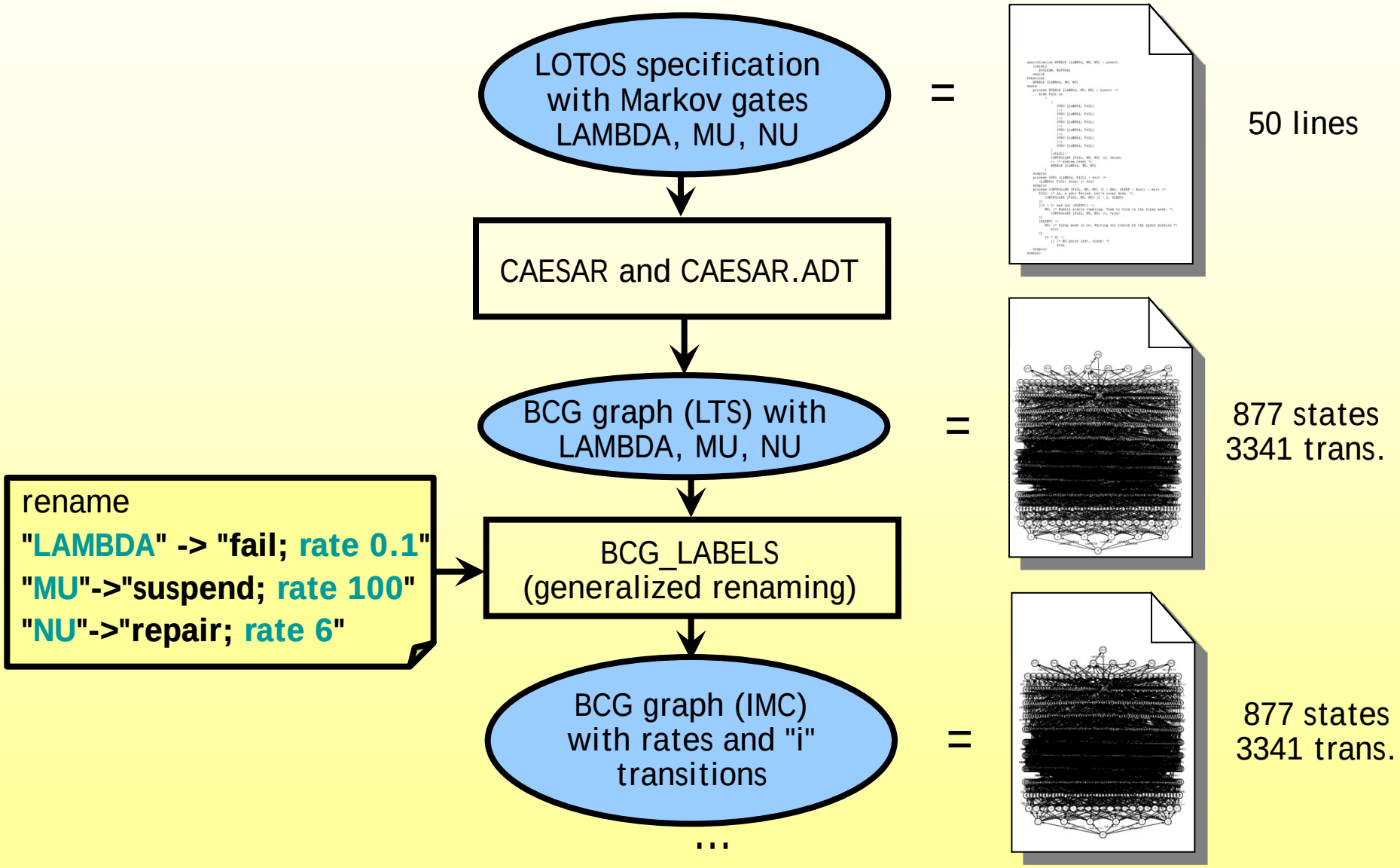



```
process GYRO [LAMBDA, FAIL] : exit :=
   (LAMBDA; FAIL; stop) [> exit
endproc
```

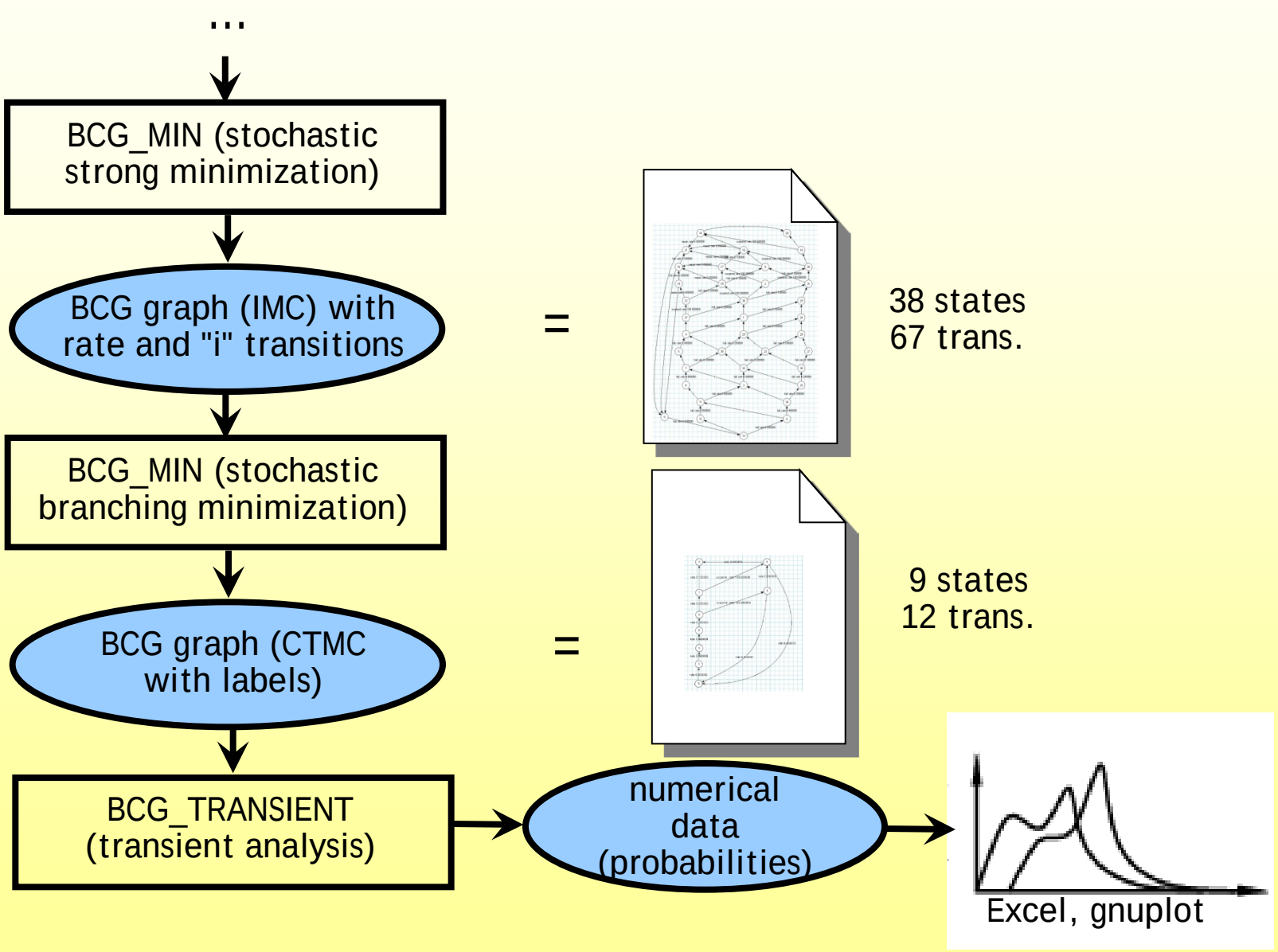
The CONTROLLER process

```
process CONTROLLER [FAIL, MU, NU] (C : Nat, SLEEP : Bool) : exit :=
    FAIL; (* Ah, a gyro failed. Let's count down. *)
        CONTROLLER [FAIL, MU, NU] (C - 1, SLEEP)
    []
    [(C < 3) and not (SLEEP)] ->
        MU; (* Hubble starts tumbling. Time to turn on the sleep mode. *)
            CONTROLLER [FAIL, MU, NU] (C, true)
    []
    [SLEEP] ->
        NU; (* Sleep mode is on. Waiting for the space mission to reset Hubble.
*)
            exit
    []
    [C = 0] ->
        i; (* No gyros left. Crash! *)
            stop
endproc
```

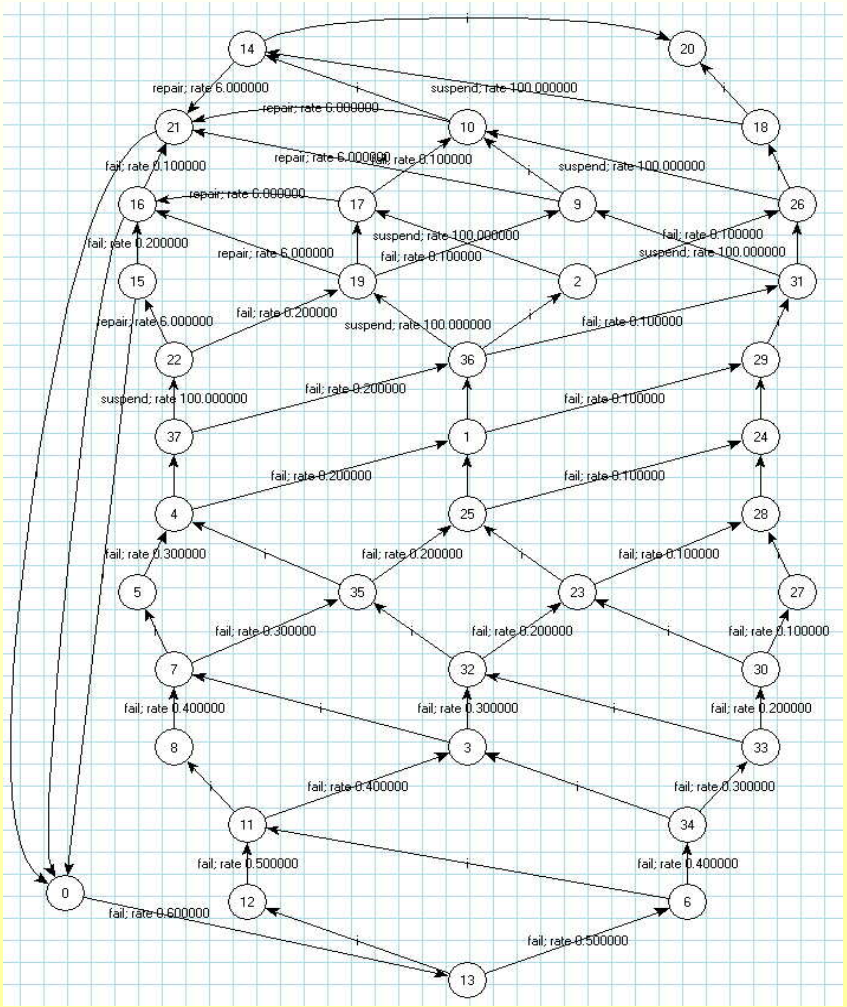
Analysis trajectory for the Hubble



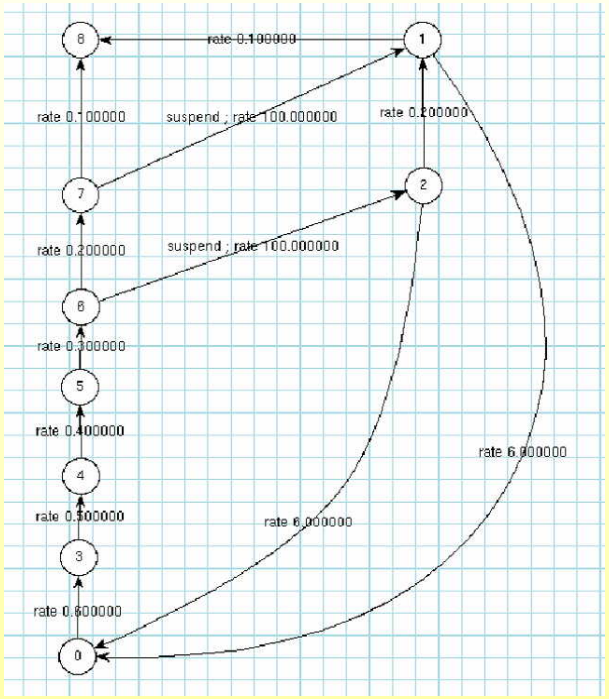
... Analysis trajectory for the Hubble



Minimized IMCs for the Hubble

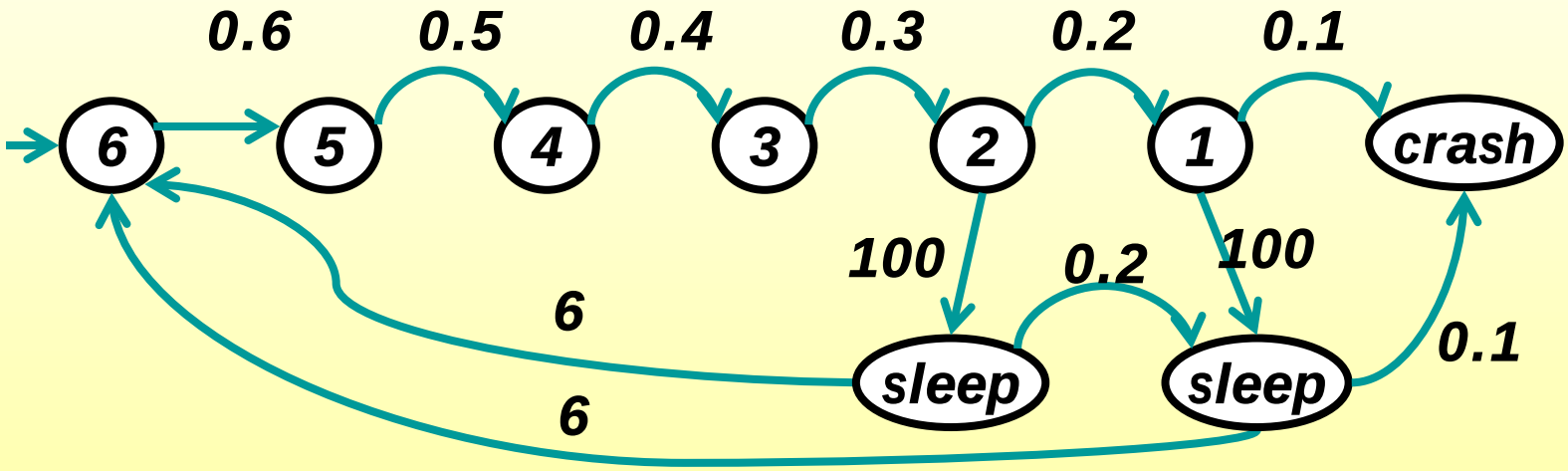


after stochastic **strong** minimization
(38 states, 67 transitions)



after stochastic **branching** minimization
(9 states, 12 transitions)

Visual verification of the final CTMC



Analysis of the Hubble using BCG_TRANSIENT

time	"repair"	"fail"	"suspend"
0.01	1.52E-11	0.5994	1.24E-09
0.1	5.45E-07	0.59403	4.34E-06
1	0.00248872	0.543138	0.00373419
10	0.105761	0.414947	0.105725
100	0.102729	0.414615	0.102786
1.00E+03	0.0974923	0.393478	0.097546
1.00E+04	0.0577739	0.233175	0.0578058
1.00E+05	0.00031195	0.00125902	0.00031212
1.00E+06	6.03E-27	2.43E-26	6.04E-27